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HEXAGON AND OCTAGON MODELS FOR EFFECTIVE THERMAL CONDUCTIVITY ESTIMATION OF POROUS GRANULAR SYSTEMS

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ABSTRACT

In this article, the geometry dependent resistance model is developed to determine the effective thermal conductivity of the unit cell-based two-phase materials. The isotherm approach is used to develop the algebraic equations for the two-dimensional spatially periodic medium. The hexagon and octagon cylinders which form the geometry of the medium are arranged in a matrix of in-line, touching and non-touching. The several two-phase materials' thermal conductivity has been estimated by the proposed models (conductivity ratio varying from 1.812 to 398.7 and concentration between 0.2 and 0.866). Two-phase materials' estimated effective thermal conductivity is compared with predicted values of standard model. Estimated values of two-phase materials have also been compared with experimental values.

KEYWORDS

Effective thermal conductivity, Concentration, Conductivity ratio and Unit-cell approach.

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INTRODUCTION

For the growth of technology and various engineering applications, such as cooling of microelectronic chips, spacecraft structures, catalytic reactors, heat recovery processes, heat exchangers, and heat storage systems, it is increasingly important to estimate the effective thermal conductivity (ETC) of two-phase materials, such as ceramics, soils, foams, emulsions, porous and suspension systems, solid-solid mixtures, fibre reinforced materials, and composites.

The subject is one that has been known for a while and has been addressed in numerous articles using the fundamental unit cell approach by taking into account both primary parameters like conductivity ratio and concentration of the dispersed phase as well as secondary parameters (contact resistance, heat transfer through radiation, Knudsen effect and geometrical configurations). To determine effective thermal conductivity of two-phase materials, a number of models were constructed. However, one of the main drawbacks of the models is their applicability for certain applications. For a variety of inclusions, the resistance model technique (Unit Cell approach) has been used to calculate effective conductivity. The most extreme limits (lower and upper) for estimating the effective thermal conductivity of two-phase materials were put forth by Maxwell[1] and Hashin-Shtrikman[2]. Wiener[3] established upper and lower limits for the conductivity of two-phase materials depending on parallel and series resistance. Zehner and Schlunder[4] proposed a model with particles in contact with each other, as well as the effect of secondary parameters. Raghavan and Martin[5] developed model to estimate the effective thermal conductivity for a random distribution of spheres in a continuum of different materials. The unit cell concept with constant heat flux constraints serve as the model's basis. Hsu *et al*[6] have used lumped parameter method, which is focused on an electric resistance analogy, to obtain algebraic equations for the effective thermal conductivities of a wide range of porous media. Samantray *et al*[7] established a comprehensive conductivity model by taking into account the primary parameters using unit cell and field solution techniques. Ultimately, the model's validity was enhanced to predict the effective conductivity of various binary metallic mixtures[8]. The collocated parameter model, based on the unit cell concept, was proposed by Reddy and Karthikeyan[9] to determine the effective thermal conductivity of the two-phase materials.

This paper's goal is to simplify the conditions by proposing extensive recommendations for choosing an applicable effective thermal conductivity model. In this paper, the primary and secondary parameters of a geometry dependent unit cell model were compared with experimental data using the unit cell approach.

Hexagon and Octagon Models for Effective Thermal Conductivity Estimation

According to Reddy and Karthikeyan[9], thermal design and analysis of two-phase systems require the development of a resistance-based unit cell model to calculate effective thermal conductivity based on material micro and nanostructure. Algebraic equations for the stationary thermal conductivity of the two-phase materials are produced using the analogy of electric resistance. The assumption of one-dimensional heat conduction in a unit cell characterizes the resistance approach.

Solids, liquids and composite layers that are normal to the temperature gradient are separated into three parallel layers that make up the unit cell. By taking into account the equivalent electrical resistances of parallel and series, the unit cell method estimates the effective thermal conductivity of a two-phase system. The thermal conductivity of the composite layer is calculated using the series model. The effective thermal conductivity of a two-dimensional medium can be estimated using an octagonal and a hexagonal cylinder with a cross-section of "a x a" and a connecting bar width of "c," as shown in Figure No.1[a] - 2 [a]. The finite contact between the cylinders created by joining the plates with the contact parameter denoted by 'c/a' induces the stagnant thermal conductivity of the two-dimensional medium. Because of the symmetry of the plates, the cross-sectional area represented in Figure No.1[b]-2 [b] is one-fourth of the cross-section, which has been treated as a unit cell. The unit cell consists of three rectangular layers that are each oriented in the direction of heat flow. Based on a series model, the solid and fluid layer's thermal conductivity is determined. The first rectangular layer of the octagon cylinder is completely occupied by the solid, having a dimension of $\left(\frac{l}{2}\right) * \left(\frac{c}{2}\right)$, while the remaining two rectangular layers are made up of the solid and fluid phases, having dimensions of $\left(\frac{l}{2}\right)\left\{\left[\frac{a}{2}+\frac{a}{\sqrt{2}}\right]-\frac{c}{2}\right\}$ and $\left(\frac{l}{2}\right)\left\{\frac{l}{2}-\left(\frac{a}{2}+\frac{a}{\sqrt{2}}\right)\right\}$respectively.The total resistance offered by the

octagon cylinder in the unit cell is determined by equation (1). The non-dimensional thermal conductivity of the two-dimensional octagon cylinder is given by equation (2). Similar to a octagon cylinder, the first rectangular layer of hexagon cylinder has a dimension of $\left(\frac{l}{2}\right) * \left(\frac{c}{2}\right)$ and is totally formed with a solid, while the other two rectangular layers, which are formed with a mixture of solid and fluid phases, have dimensions of $\left(\frac{l}{2}\right)\left[a\frac{\sqrt{3}}{2} - \frac{c}{\sqrt{2}}\right]$and $\left(\frac{l}{2}\right)\left[\frac{l}{2} - a\frac{\sqrt{3}}{2}\right]$ respectively. The non-dimensional thermal conductivity of the two-dimensional hexagonal cylinder is given by equation (3).

Equations

$$\boldsymbol{R_{Total}} = \frac{\boldsymbol{\varepsilon\lambda}}{\boldsymbol{\alpha}} + \left[\frac{\frac{1}{\alpha}\left[\left(\frac{(1-\lambda)}{1+\sqrt{2}}\right)+\frac{2\sqrt{2}}{2+\sqrt{2}}\right]*\frac{2\varepsilon\,([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f}\{[2(1-(\varepsilon+\varepsilon\sqrt{2}))]+[\varepsilon(1-\lambda)+\sqrt{2}]\}}}{\frac{1}{\alpha}\left[\left(\frac{(1-\lambda)}{1+\sqrt{2}}+\frac{2\sqrt{2}}{2+\sqrt{2}}\right)\right]+\frac{2\varepsilon\,([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f}\{[2(1-(\varepsilon+\varepsilon\sqrt{2}))]+[\varepsilon(1-\lambda)+\sqrt{2}]\}}}\right]+ \left[\frac{[1-(\varepsilon+\varepsilon\sqrt{2})][\alpha+(1-\alpha)\varepsilon\lambda]}{\alpha\{(1-\varepsilon\lambda)+\varepsilon\lambda\}[\alpha+(1-\alpha)\varepsilon\lambda]}\right] \quad (1)$$

$$K = \frac{k_{eff}}{k_f} = \frac{\boldsymbol{\varepsilon\lambda}}{\boldsymbol{\alpha}} + \left[\frac{\frac{1}{\alpha}\left[\left(\frac{(1-\lambda)}{1+\sqrt{2}}\right)+\frac{2\sqrt{2}}{2+\sqrt{2}}\right]*\frac{2\varepsilon\,([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f}\{[2(1-(\varepsilon+\varepsilon\sqrt{2}))]+[\varepsilon(1-\lambda)+\sqrt{2}]\}}}{\frac{1}{\alpha}\left[\left(\frac{(1-\lambda)}{1+\sqrt{2}}+\frac{2\sqrt{2}}{2+\sqrt{2}}\right)\right]+\frac{2\varepsilon\,([1-\lambda]+\sqrt{2})}{\frac{k_{sf2}}{k_f}\{[2(1-(\varepsilon+\varepsilon\sqrt{2}))]+[\varepsilon(1-\lambda)+\sqrt{2}]\}}}\right]+ \left[\frac{[1-(\varepsilon+\varepsilon\sqrt{2})][\alpha+(1-\alpha)\varepsilon\lambda]}{\alpha\{(1-\varepsilon\lambda)+\varepsilon\lambda\}[\alpha+(1-\alpha)\varepsilon\lambda]}\right] \quad (2)$$

Where,

$$\frac{k_{sf2}}{k_f} = \frac{\alpha\left(\left[\frac{1-\lambda}{2}\right]+\frac{1}{\sqrt{2}}\right)}{\left[\varepsilon\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)(2-\lambda)+\alpha\left[\frac{(1-\lambda)}{2}+\frac{1}{\sqrt{2}}\right]-\left[\varepsilon\alpha\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)(2-\lambda)\right]\right]}$$

$$K = \frac{k_{eff}}{k_f} = \frac{\boldsymbol{\varepsilon\lambda}}{\boldsymbol{\alpha}} + \left[\frac{\frac{2\sqrt{3}\varepsilon(\sqrt{3}-\lambda)}{\left[(\sqrt{3}-\lambda)+2\sqrt{3}+\frac{k_{sf2}}{k_f}*[2\sqrt{3}]\left(1-\left(\varepsilon+\left(\varepsilon-\frac{\varepsilon\lambda}{\sqrt{3}}\right)\right)\right)+\varepsilon(\sqrt{3}-\lambda)\right]}}{\frac{1}{[(\sqrt{3}-\lambda)+2\sqrt{3}]}+\frac{\varepsilon}{\left[(\sqrt{3}-\lambda)+2\sqrt{3}*\frac{k_{sf2}}{k_f}*[2\sqrt{3}]\left(1-\left(\varepsilon+\left(\varepsilon-\frac{\varepsilon\lambda}{\sqrt{3}}\right)\right)\right)+\varepsilon(\sqrt{3}-\lambda)\right]}}\right]+ \frac{\frac{(1-\varepsilon\sqrt{3})}{\frac{k_{sf3}}{k_f}[\varepsilon\lambda(1-\varepsilon\lambda)]}}{\left[\frac{1}{\frac{\varepsilon\lambda}{\alpha}+\frac{1}{\left\{\frac{1}{\alpha}+\left(\frac{k_{sf3}}{k_f}\right)(1-\varepsilon\lambda)\right\}}}\right]} \quad (3)$$

Where

$$\frac{k_{sf3}}{k_f} = \frac{\alpha}{\left[\alpha+\left(\varepsilon+\left(\frac{\varepsilon}{2}-\frac{\varepsilon\lambda}{2\sqrt{3}}\right)*(1-\alpha)\right)\right]}$$

RESULTS AND DISCUSSION

The effective thermal conductivity of a two-phase system is dependent on the size, shape, thermal contact, and solid-solid and solid-fluid interfaces as well as the thermal conductivities of the solid and fluid phases. To determine the effective thermal conductivity of different inclusions on granular systems, a geometry dependent effective thermal conductivity has been created. For the porous granular material, a comparison has been made between the suggested model and experimental data given by Reddy and Karthikeyan[9] over a range of concentrations. For porous granular materials, the octagon cylinder showed good agreement with the experimental results (υ = 0.2 to 0.866 and α = 1.812 to 398.7). The accuracy range appears to be fairly good when you consider the range of data sources and geometries. The average deviation from experimental data for the octagon cylinder is observed to be 10.25%, whereas it is 14.12% for the hexagon cylinder.

NOMENCLATURE

a -Length of the hexagon and octagon cylinders
c -Width of the connecting plate in the hexagon and octagon cylinders
K - Non-dimensional thermal conductivity of the two-phase materials (keff / kf)
keff - Effective thermal conductivity of two-phase materials, (W/mK)
kf - Fluid or continuous thermal conductivity, (W/mK)

ks - Solid or dispersed thermal conductivity, (W/mK)
ksf - Equivalent thermal conductivity of a composite layer, (W/mK)
R - Thermal resistance, (m 2 K/W)
l - Length of the unit cell, (m).

Table No.1: Comparison of present geometry dependent model with experimental data for porous granular systems

υ	α	Kexp	λ	Khex	Devi. (%)	Koct	Devi. (%)	System/Source [8]
0.2	45.79	1.708	0.01	2.055	20.33	2.032	18.99	Glasssphere/air
0.41	56.96	4.06	0.02	4.241	4.46	4.078	0.44	Wassauand/helium
0.456	127.47	7.34	0.03	8.674	18.18	7.681	4.65	Miamisiltfoam/air
0.47	95.285	5.714	0.02	6.324	10.68	5.919	3.59	Zirconapowder/air
0.485	64.91	5.596	0.02	5.572	0.43	5.374	3.96	Wassausand/n-heptane
0.495	61.91	5.9614	0.03	6.492	8.89	6.120	2.66	Stainlesssteel/eth.alcohol
0.507	128.6	8.879	0.03	10.377	16.87	9.280	4.52	Air/calcite
0.511	138.1	6.328	0.01	6.563	3.71	6.345	0.27	He/steel
0.535	19.7	4.051	0.01	3.951	2.46	4.225	4.31	Etoh/calcite
0.547	6.1	2.515	0.01	2.710	7.76	3.211	27.66	H_2O/silica
0.552	127.47	9.6	0.02	9.802	2.10	9.161	4.58	Miamisiltfoam/air
0.56	398.7	15.336	0.01	13.947	9.06	12.669	17.39	Air/quartz
0.561	17.9	3.963	0.01	4.127	4.15	4.477	12.96	H_2O/silica
0.563	16	5.244	0.08	5.193	0.97	5.292	0.91	Air/Coal
0.563	2.2	1.524	0.9	1.764	15.73	1.995	30.91	H_2/coal
0.563	16	5.23	0.07	5.045	3.53	5.176	1.04	Air/coal
0.563	2.17	1.53	0.9	1.746	14.15	1.977	29.23	H_2/coal
0.569	21.18	4.341	0.01	4.440	2.28	4.789	10.32	Silicasphere/water
0.569	17.868	4.494	0.01	4.218	6.14	4.590	2.14	Water/silica
0.569	7.648	2.859	0.01	3.113	8.88	3.630	26.95	IC8/glass
0.57	7.368	2.8194	0.01	3.071	8.91	3.598	27.60	Glasssphere/iso-octane
0.572	2.03	1.5832	0.01	1.579	0.24	2.224	40.45	Glycerin/glass
0.575	104.37	5.724	0.01	7.389	29.08	7.422	29.67	H_2/SiC
0.575	104.4	5.7	0.01	7.390	29.64	7.423	30.23	H_2/SiC
0.576	290.5	9.876	0.01	12.199	23.52	11.413	15.57	Air/SiO
0.577	3.023	1.891	0.01	2.001	5.82	2.665	40.95	Etoh/glass
0.58	66.7	7.66	0.02	7.761	1.32	7.679	0.25	Zirconapowder/air
0.58	7.824	2.862	0.01	3.216	12.38	3.762	31.45	He/glass
0.58	2.06	1.572	0.9	1.696	7.88	1.942	23.53	Glycerol/glass
0.58	1.812	1.384	0.9	1.546	11.68	1.782	28.75	H_2O/glass
0.6	57.617	7.387	0.01	7.816	5.80	7.877	6.63	Lead/Water
0.6	37.62	6.206	0.01	5.806	6.45	6.191	0.24	Glassbeds/air
0.6	43.46	6.769	0.02	6.912	2.11	7.093	4.78	Glass/air

0.6	124.2	7.213	0.01	8.746	21.25	8.788	21.84	Glycerin/lead
0.6	161.4	8.86	0.01	9.839	11.05	9.715	9.65	Air/sand
0.603	191.1	8.025	0.01	10.833	34.99	10.585	31.90	Etoh/lead
0.612	253.3	12.775	0.01	13.142	2.87	12.642	1.04	Glycerin/Cu
0.612	253.3	12.8	0.01	13.142	2.67	12.642	1.23	Cu/glycerolsolution
0.62	233.65	14.55	0.01	13.013	10.56	12.652	13.04	Leadshots/helium
0.62	191.88	13.569	0.01	11.697	13.79	11.536	14.99	Leadshots/hydrogen
0.62	54.77	8.618	0.02	8.247	4.30	8.422	2.27	Lead shots/water
0.639	7.864	3.398	0.01	3.674	8.12	4.451	30.98	Microbeads/soltrol
0.64	66.7	9.36	0.02	9.834	5.06	10.031	7.17	Zirconapowder/air
0.64	56.96	9	0.02	9.106	1.18	9.385	4.28	Ottawasand/Helium
0.65	42.89	7.857	0.01	7.320	6.84	7.988	1.66	Glassbeds/air
0.65	8.578	3.571	0.01	3.935	10.19	4.756	33.18	Glassbeads/benzene
0.65	40.23	7.423	0.01	7.165	3.47	7.838	5.60	Micro beads/air
0.655	9.4	5.7	0.1	4.855	14.82	5.534	2.90	Air/Cr/Alcatalyst
0.655	11.6	5.8	0.05	5.097	12.11	5.792	0.14	Air/Cr/Alcatalyst
0.676	8.069	3.759	0.01	4.068	8.21	5.053	34.42	Quartzsand/Water
0.7	66.7	12.13	0.01	10.859	10.48	12.189	0.49	Ziconapowder/air
0.7	6.8	4.2	0.01	3.914	6.81	5.076	20.86	Air/Pt/Al_2O_3/catalyst
0.71	7.8	4.45	0.01	4.351	2.23	5.600	25.84	Air/Co/Mo catalyst
0.725	8.1	6.6	0.2	5.431	17.71	6.589	0.17	Behmite
0.74	45.79	9.458	0.01	11.686	23.56	14.064	48.70	Glasssphere/air
0.77	14.5	9.8	0.01	7.697	21.46	10.115	3.22	Air/Ni/Wcatalyst
0.866	8.1	8.3	0.9	6.223	25.03	8.459	1.91	Powder
Average Deviation (%)				14.12		10.25		

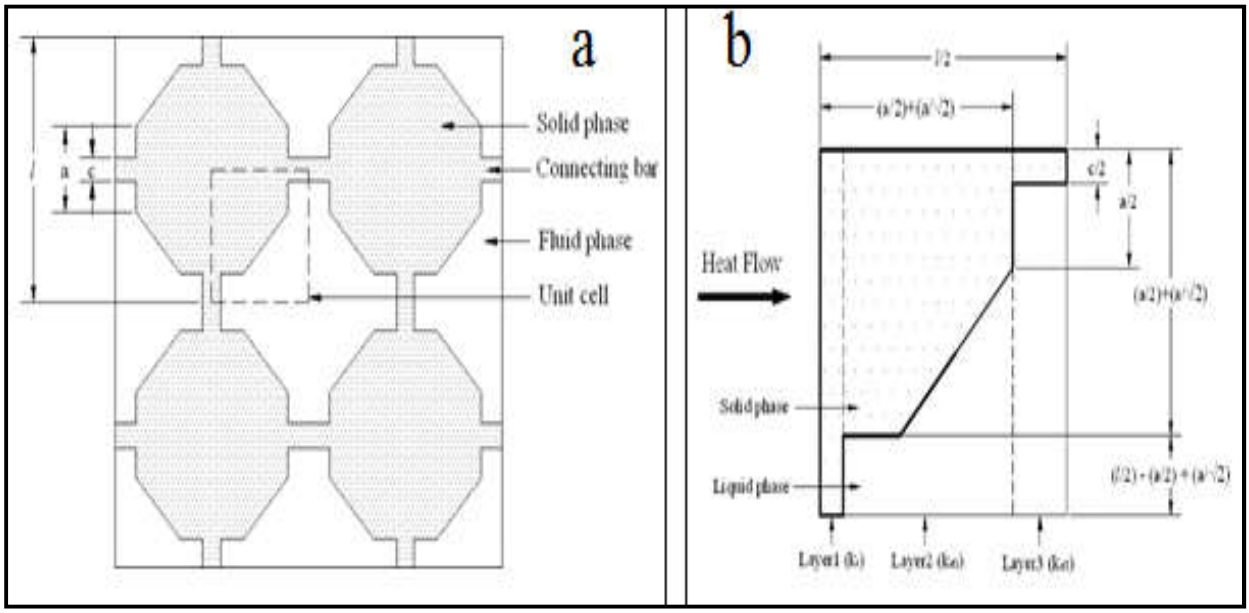

Figure No.1: Two-dimensional spatially periodic two-phase system: (a) Touching Octagonal cylinder (b) Unit cell of Octagonal cylinder

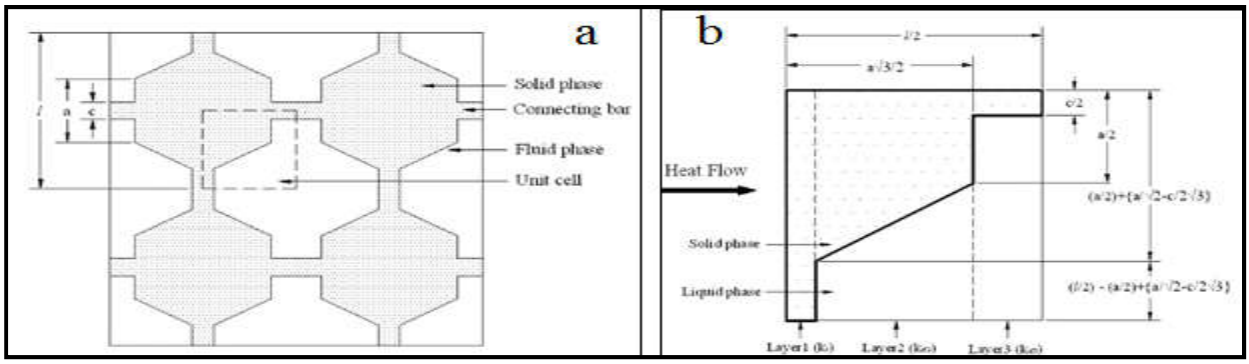

Figure No.2: Two-dimensional spatially periodic two phase system: (a) Touching Hexagon cylinder (b) Unit cell of Hexagonal cylinder

CONCLUSION

Geometry dependent models are formed with the influence of hexagon and octagon cylinders in order to calculate the effective thermal conductivity of two-phase materials. The experimental data are also used to validate the current models. The effective thermal conductivity of two-phase materials used in engineering applications can be estimated using the current models.

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CONFLICT OF INTEREST

We declare that we have no conflict of interest.

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